SAMPLE MATERIAL

Curriculum Guide for Gifted Students

Longfellow Middle School, Virginia

Topic: National Math Panel: Critical Foundations for Algebra
Practice: Mastery Framework

This curriculum guide lists the topics that are addressed during the Longfellow Middle School course designed for gifted seventh-grade students. While the course evolves and uses a variety of materials, the guide lists the topics that are typically covered within major categories such as Cantorian Set Theory; Exponents, Radicals, Scientific Notation, Bases, and Compound Unit Conversion; Geometry; Sequences, Series, Induction, and Basic Calculus.

Math 7 Honors(Longfellow's Version)

Honors Math, formally known as Math 7 GTC(only at Longfellow) was developed in order to fulfill the needs of mathematically gifted seventh grade students attending Longfellow. The course involves a survey of high school math topics as well as the discussion of some topics taken from college level math courses. Problem soling, theory and abstract reasoning are stressed throughout the course and students are encouraged to develop and test their own theories and to write creative solutions to difficult math problems. The course is constantly evolving and new problems and short topics are selected from Mathematical Association of America journals, high school math contests, high school and college textbooks. MathCounts contests, students and parents. Probability and statistics are analyzed and applied by solving previous and current contest problems involving those topics. No textbook is used and the pace is accelerated.

Curriculum Guide:

Cantorian Set Theory

- Show that all of mathematics is based on the concept of set.
- Relate sets to geometry by applying union and intersection to sets of points.
- Give a rigorous definition of infinite set, i.e. use the definition of subset and the concept of one-on-one correspondence.
- Theory of infinite sets(Cantor)
- Discuss denumerable and non-denumerable sets.
- Count the rationals using Cantors diagonal method.
- Prove that the set of reals cannot be put into one-on-one correspondence with the set of natural numbers.
- Use two to the nth power to introduce higher levels of infinity and show that there are an infinite number of levels of infinity.
- Define and compare sets of numbers including complex numbers.
- Define and apply union, intersection, difference, complement, universal set, cardinal number, ordinal number, set builder notation, and quantification notation.
- Perform operations on sets.
- Use set notation in geometry and probability
- Do proofs involving sets
- Solve problems involving sets. Problems are selected from past high school math contest.

Groups

- Discuss and develop, in depth, all of the real number properties.
- Apply group properties to operation tables and discuss mathematical systems.
- Develop an axiomatic system for proving group theorems.
- Discuss permutation groups and solve related problems.

- Simplify algebraic expressions using properties and order of operations.
- Translate between group, multiplicative and additive languages.
- Prove basic field theorems such as the multiplicative and additive properties of
- Equality and the Zero property.
- Perform operations using modular arithmetic.
- Use modular arithmetic to solve problems such as the international Mathematics Olympiad problem involving infinite sequences and mod eight.

Rationals

- Perform operations on rationals.
- Give an informal proof of the density property.
- Use the formula a/(1-r) to add an infinite series of rationals.
- Review field properties.
- Convert repeating decimals to common fractions
- Simplify rational expressions involving variables
- Discuss attempted division by zero
- Simplify complex fractions
- Use the Euclidian Algorithm and the unique factorization theorem to simplify rational expressions.
- Discuss Skewes number and relate it to the infinite number of primes proof.
- Prove that the number of factors for (2 to the a)(3 to the b)(5 to the c)… =(a+1)(b+1)(c+1)

Exponents, Radicals, Scientific Notation, Bases, Compound Unit Conversion

- Derive and apply laws of exponents including fractional exponents.
- Simplify radicals, both numerical and involving variables
- Perform operations on radicals
- Reduce the index of a radical
- Rationalize denominators involving radicals
- Give a geometric derivation of the square root algorithm
- Solve problems using scientific notation
- Work with different number bases including negative, fractional and radical
- Perform operations within different number bases
- Relate number bases to computer technology
- Use properties of reals to perform compound unit conversion
- Solve distance rate time problems using compound unit conversion
- Solve geometry and capacity problems using compound metric conversion

Geometry

- Compare and contrast Euclidian geometry with the geometries of Lobachevski, Bolyai, Riemann

- Define and discuss, projective, analytic, transformational, abstract and differential geometry
- Discuss angles and angle theorems including angles of a circle, radian measure and angles in trigonometry.

- Introduce undefined terms, basic theorems and do basic proofs such as the vertical angle theorem, some congruent triangle proofs, and the sum of the measures of the angles of a Euclidian triangle theorem
- Present multiple derivations of the Pythagorean Theorem
- Discuss similarity theorems (no proof) and apply them to solve problems
- Discuss and apply circle power theorems(no proof)
- Derive formulas to compute the number of diagonals and the interior and exterior angles of a polygon
- Review the properties of quadrilaterals
- Derive and apply perimeter, area, and volume formulas including special formulas, i.e. the area of an equilateral triangle
- Introduce and apply Herons formula and present a special after school derivation
- Solve complex geometry problems taken from high school math contests, Mathcounts, the **The Art of Problem Solving.**

Mathematical Logic

- Construct truth tables, both simple and complex
- Design and simplify logic circuits to solve problems involving electric currents
- Discuss rules of inference such as modus ponens and logical fallacies such as false cause.
- Relate Boolean logic to computer programming and design
- Use truth tables and rules of inference to analyze arguments
- Do proofs using truth tables
- Compare and contrast implication, inverse, converse and contrapositive
- Relate logic to language arts by analyzing the structure of clauses
- Review basic geometry proofs using the rules of mathematical logic
- Compare and contrast Boolean(symbolic)logic with Aristotelian(syllogistic) logic

Equations and Algebraic Word Problems

- Solve all equations in one variable except quadratics with non-rational roots, non-simple radical equations and rational expressions involving polynomial factoring
- Solve linear systems in two, three or more variables using linear combinations, substitution, determinants, graphing and matrices(Gaussian Elimination)
- Derive the above methods including Cramer's rule(determinants)
- Solve the following types of word problems:
  Number and digit problems

Consecutive integer problems
Mixture problems
Distance rate time problems
Percent problems
Geometry problems(using algebraic methods)
Ratio and proportion problems
Student created problems
Algebraic word problems taken from the American High School Math Exam

Curve Sketching (Analytic Geometry)

- Recognize the slope and intercepts from the equation of a line
- Write the equation of a line given two points
- Prove that m represents the slope when y = mx+b
- Recognize through derivations or tables the equation of a parabola, circle, ellipse, hyperbola, cubic, absolute value, greater integer function, exponential function and a selection of non-standard curves.
- Derive sketches for sine, cosine, tangent and their reciprocals via the unit circle.
- Discuss the concept of function
- Sketch curves using polar coordinates
- Compare and contrast Cartesian,, polar, cylindrical and spherical coordinates
- Discuss asymptotes and use them to aid in curve sketching
- Manipulate equations to shift curves horizontally and/or vertically
- Use the Green Globs computer software to confirm and extend comprehension of curve sketching

Trigonometry

- Compute measures of sides and angles of right triangles
- Use scientific or graphing calculators to solve right and oblique triangles
- Use the law of cosines to fine measures of sides and angles of oblique triangles
- Give a special after school derivation of the law of cosines
- Define and apply angles of elevation and depression
- Solve complex word problems involving right and oblique triangles

Sequences, Series, and an Introduction to Induction and Basic Calculus

- Derive formulas for finding the nth term of an arithmetic or a geometric sequence
- Derive formulas for finding the sum of an arithmetic or a geometric series
- Discuss infinite geometric sequences and limits
- Use the ratio test to find the sum of an infinite geometric series
- Discuss and apply arithmetic and geometric means
- Discuss and compute limits using the division of powers method

- Derive the formula for determining the slope of a curve at a given point and apply it to basic curves such as parabolas and cubics
- Use the power rule to find derivatives of basic functions
- Discuss scientific and engineering applications of calculus
- Give a general discussion of induction
- Prove, via induction, that the sum of the first N odd numbers equals N to the second power.
- Discuss and use integration to compute the area under basic curves such as parabolas and cubics.